MATHEMATICAL REGRESSION MODEL OF UNIDIRECTIONAL GLASS FIBRE REINFORCED POLYMER COMPOSITES

Emilia SABAU[1], Adrian POPESCU[1], Cristina-Ștefana Miron-Borzan[1] and Nicolae PANC[1]
[1] Technical University of Cluj-Napoca, Machine Building Faculty, Department of Manufacturing Engineering, 103-105 Muncii Bd., 400641, Cluj-Napoca, Romania, E-mail: Emilia.Sabau@tcm.utcluj.ro

***ABSTRACT**: A mathematical regression model for prediction of the compression behavior of unidirectional fibres glass reinforced polymer composites is described in this paper. The composites plates were obtained through hand lay-up technology. The experiments have consisted of compression tests, where the glass fibers were inclined at angles of 0º, 45º and 90º in report with the load direction. A polynomial regression model has been applied to establish the dependence of the compression resistance on theoretical-experimental way. The data were analyzed using SigmaPlot 10.0 software that performs these calculations. A close agreement has been obtained between the predicted and experimental results.*
***KEYWORDS**: composite materials, unidirectional glass fibre, polymer matrix, mathematical model, polynomial regression*

1 INTRODUCTION

The importance of parts realized form composite materials (CM) are growing everyday, thanks to performances/cost characteristics and excellent mechanical properties reported at weight.

In theoretical and experimental research cases is not enough to obtain only numerical values, to identify the mechanisms of failure: fibre failure, elastic micro buckling, matrix failure and so on, (Jelf, 1992). This must to be analyzed and interpreted. Therefore, in the literature are a number of references about the use of regression analysis like a statistical technique for estimating the relationship among variables which have reason and result relation, (Uyanık, 2013).

Regression analysis involves identifying the relationship between a dependent variable and one or more independent variables. A model of the relationship is hypothesized, and estimates of the parameter values are used to develop an estimated regression equation.

Model validation is an important step in the modelling process and helps in assessing the reliability of models before they can be used in decision making, (Ostertagová, 2012).

A mathematical model for prediction of the compression behavior of woven reinforcement is described by (Lomov, 2000). The model accounts for the compression of yarns and the change of yarn crimp under pressure applied to a fabric layer.

Another, mathematical model has been developed by (Palanikumar, 2006) to predict the surface roughness of machined glass fiber reinforced polymer (GFRP) work piece using regression analysis in order to study the main and interaction effects of machining parameters, viz., cutting speed, work piece fiber orientation angle, depth of cut, and feed rate.

(Palanikumar, 2007) developed a mathematical model to predict the tool wear on the machining of GFRP composites using regression analysis in order to study the main and interaction effects of machining parameters, viz., cutting speed, feed rate, depth of cut and work piece fibre orientation angle.

Mathematical regression model have been developed to predict theoretical wear rate of the composite by (Satyanarayana, 2018).

Cubic spline of regression with application at composite materials was presented by (Sabău et al., 2009).

The machining forces-tool wear relationship of an aluminum metal matrix composite has been studied by (Lin, 2003) using multiple regression analysis (MRA)

(Mavromatidis, 2013) propose a simple method based on classic and fractional factorial simulation plans to obtain regression models in the form of polynomial functions that link the angle, the thermal conductivity and the thickness of each envelope's component to the overall wall's thermal resistance.

The polynomial regression model has been applied by (Ostertagová, 2012) using the characterization of the relationship between strains and drilling depth. Parameters of the model were estimated using a least square method.

In (Eswari, 2011) is presented regression equations for evaluating the strength and toughness of hybrid fibre reinforced concrete. Empirical expressions for predicting the strength and toughness of hybrid fibre reinforced concrete are proposed based on regression analysis.

Taking into consideration the previous researches in the field, the purpose of this paper is to establish the dependence of the compression resistance on theoretical-experimental way, using the regression analysis of the active experiment.

The compression strength of unidirectional glass fibre reinforced polymeric (UD-GFRP) composites has been examined.

Regression analysis is one of the most important statistical tool that is extensively used in almost all sciences.

Regression modeling was used to study the dependence between reinforcement grades, fibres orientation angle and compression strength of the UD-GFRP.

Mathematical regression model have been developed to predict theoretical compression strength of the composite.

2 MATERIALS AND METHOD

2.1 Materials

The materials used in this study are unidirectional (UD) glass fibres and polyester resin.

Polyester resin was used as matrix. The type and the characteristics of this material are: Polyester resin type Lerpol TIX 3603/R (Italy) is ortophtalic, tixotropized and ecological.

Technical characteristics of the resin are: Color: blue; Brookfield viscosity at 25°C: 400 cPs; Density at 16/15ºC: 1530 kg/m^3, Time for obtaining jelly with 2 % peroxide of MEC (methyl-ethyl-ketone) in 100g resin: 12 min; Exothermic peak: 185ºC; Exothermic peak time: 19’30” min; Solubility in water at 25ºC: 130 mg/l.

The fabric is unidirectional glass fibres 225 g/m^2 (Germany): Warp: Roving 300 tex; Weft: Garn 22 tex.

2.2 Manufacturing methodology

The hand lay-up technique is the most used and oldest method of composite material parts manufacturing.

The composite plates used during the experiments and the tests were performed in the Laboratory of the Technical University of Cluj-Napoca.

The applied technology was hand lay-up. The manufacturing process of obtaining the composite plates was performed at ambient temperature of 20°C.

Theoretical calculations were made regarding the mechanical characteristics of the elaborated plates and were made samples to determine the physical and mechanical characteristics at compression tests.

Different plates with different reinforced grade were obtained. From obtained composite material five samples were cut. The fibres were inclined at angles of 0º, 45º and 90° in report with the trial direction. These have 10 mm x 4.36 mm x 60 mm dimensions according to manufacturing and testing standard SR EN ISO 14126:2003.

To determine the mechanical characteristics of composite plates was done experimental trials at compression.

The testing universal machine was Instron 1196, and the speed for compression tests was 2 mm/min.

The resistance at compression tests depends on reinforcement grade and angle between fibers direction in UD composite and stress direction.

For compression breaking strength (σ_{rc}) determination the maximum force has been reported at cross-sectional area, using the relation:

$$\sigma_{rc} = \frac{F_{\max}}{A} \text{ [MPa]} \quad (1)$$

where: $F_{\max}$ - maximum compression load at the sample fracture, [N]; A - initial cross-sectional area of the sample, [mm^2].

2.3 Polynomial regression model

The laminated composites have a different compression behavior compared to the tensile behavior, because of the different way of fibers request (Sabău et al., 2013).

At compression tests the laminates shows failed because of transverse buckling or shearing of the fibers. It was considered to establish the dependence of the compression strength on the reinforcement degree and the orientation angle of the reinforced material.

For mathematical modeling of the structure influence on compression strength of UD composite was utilized a regression analyze that used a polynomial model. The nonlinear mathematical model can be expressed:

$$\sigma_{rc} = a_0 + a_1 M_f + a_2 \theta + a_{11} M_f^2 + a_{12} M_f \theta + a_{22} \theta^2 \quad (2)$$

where: σ_{rc} - compression strength of unidirectional composite [MPa] – objective function; θ - angle between fibers direction in composite and stress direction [deg.] – independent variable; M_f – reinforcement degree [%] - independent variable; a_0, a_1, a_2, a_{11}, a_{12}, a_{22} - unknown coefficients.

3 RESULTS AND DISCUSSION

The obtained experimental results for UD-GFRP composite structure made from 16 layers, with UD glass fibres 225 g/m^2 and polyester resin Lerpol TIX 3603/R, are presented in Table 1.

The compression breaking strength is directly influenced by the number of reinforcement layers, because the fibers take the internal forces due to the mechanical stresses.

Table 1. Experimental results

No. Exp.	θ [grd.]	M_f [%]	σ_{rc} [MPa]
1.	0º	51.33	298.2
2.	0º	62.33	314.0
3.	0º	70.66	368.4
4.	45º	51.33	109.8
5.	45º	62.33	115.2
6.	45º	70.66	130.6
7.	90º	51.33	114.4
8.	90º	62.33	108.4
9.	90º	70.66	106.2

The compression strength has a maximum of 368.4 MPa, depending on reinforcement grade and angle between fibers direction in UD composite and stress direction.

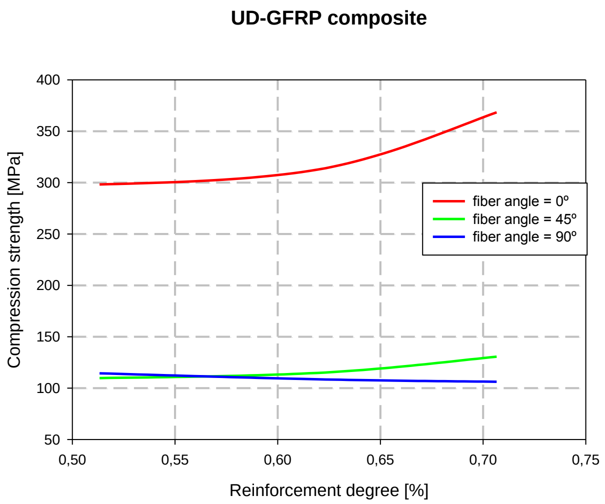


Fig. 1 Compression strength in function of reinforcement degree for UD composite

In charts from Figure 1 and Figure 2, the compression strength variation in function of reinforcement degree is show, respectively the compression strength in function of fibres orientation from UD composites.

It is observed that, the UD composite with fibers inclined at angle of 0º have higher mechanical characteristics then composite with fibers inclined at angles of 45° and 90º.

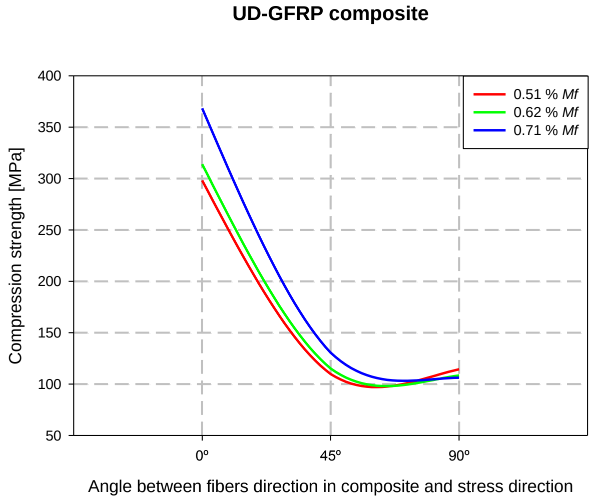


Fig. 2 Compression strength in function of fibres orientation from UD composite

In the case of UD composites the forces are taken in the longitudinal direction of the fibers.

For mathematical model the average values were used.

Using the SigmaPlot 10.0 software, the reports of the compression strength of the UD composite according to the structural characteristics were obtained.

Table 2. Values of the multiple correlation coefficient, R square, Adj. R square and Std. error of estimate

R	Rsqr	Adj Rsqr	Standard Error of Estimate
0.9988	0.9976	0.9937	8.5834

For UD glass fibre 225 g/m^2 / polyester resin Lerpol TIX 3603/R with 16 layers, the multiple correlation coefficient (R) has the value 0.9988 and R square value is 0.9976, which confirms the validity of the model, Table 2.

Table 3. Values of coefficients and standard error, t-value and p-value

Coef.	Estimates	Std. Error	t-value	p-value
a0	542.5499	243.8559	2.2249	0.1125
a1	-1076.2928	809.7363	-1.3292	0.2758
a2	-4.1508	0.6664	-6.2289	0.0083
a11	1161.1360	664.4804	1.7474	0.1789
a12	-4.3865	0.9836	-4.4595	0.0210
a22	0.0493	0.0030	16.4323	0.0005

From the generated reports of the program, the unknown coefficients for mathematical model were obtained, Table 3. Also, standard error, t-value and p-value for the coefficients of the model are furnished.

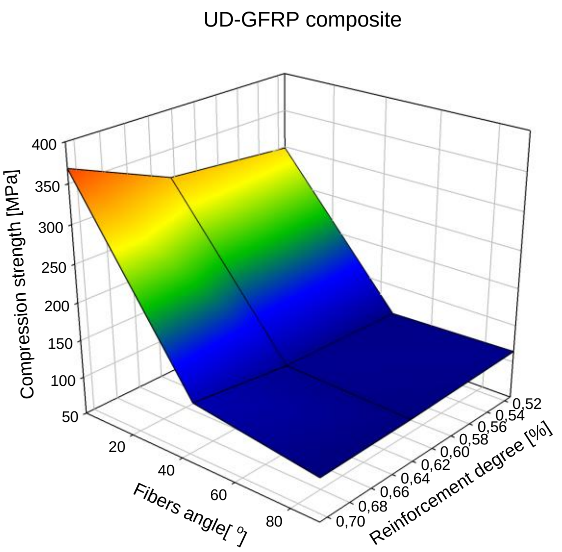
The empirical values obtained for the model are:

$a_0 = 542.5499$, $a_1 = -1076.2928$, $a_2 = -4.1508$,
$a_{11} = 1161.1360$, $a_{12} = -4.3865$, $a_{22} = 0.0493$

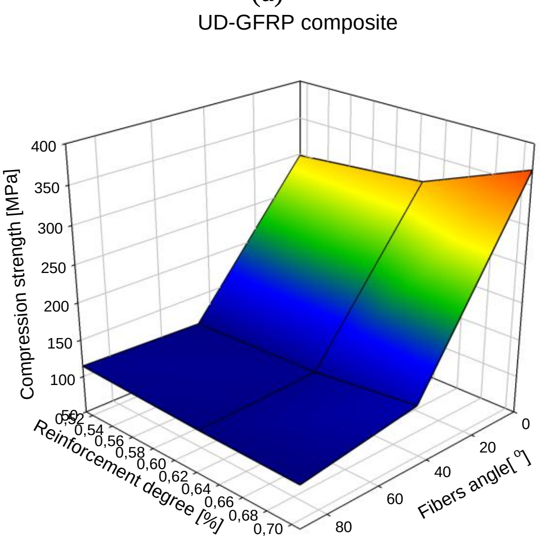
The obtained mathematical model for compression strength of studied composite material has the following form:

$$\sigma_{rc} = 542.55 - 1076.29\, Mf - 4.15\, \theta + 1161.14\, Mf^{\,2} - 4.39\, Mf\, \theta + 0.05\, \theta^2 \quad (3)$$

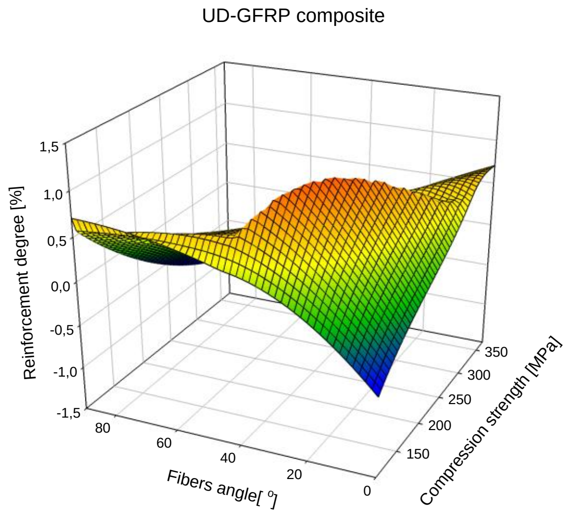
This was graphically represented with the help of SigmaPlot 10.0. software, figure 2(a), (b) and (c).



(a)



(b)



(c)

Fig. 3 Graphical representation of the mathematical model of compression strength

Due to the mathematical model presented above, the compression strength of the UD laminate in function of the reinforcement degree of the glass fibers and the reinforcement angle was calculated.

The both structural characteristics varied in the intervals where the mathematical modeling was done, $M_f \in [50\ldots70]$ % and $\theta \in [0°\ldots90°]$.

The predicted values using regression model are presented in Table 4.

Table 4. Predicted values using regression model

No. Exp.	θ [grd.]	M_f [%]	σ_{rc} [MPa]
1.	0º	51.33	296.0213
2.	0º	62.33	322.8012
3.	0º	70.66	361.7775
4.	45º	51.33	107.6444
5.	45º	62.33	112.7111
6.	45º	70.66	135.2444
7.	90º	51.33	118.7343
8.	90º	62.33	102.0876
9.	90º	70.66	108.1781

A comparative study was made between experimental data and values from regression model, Figure 4.

It is observed that the predicted values using regression model have excellent agreement with measured values.

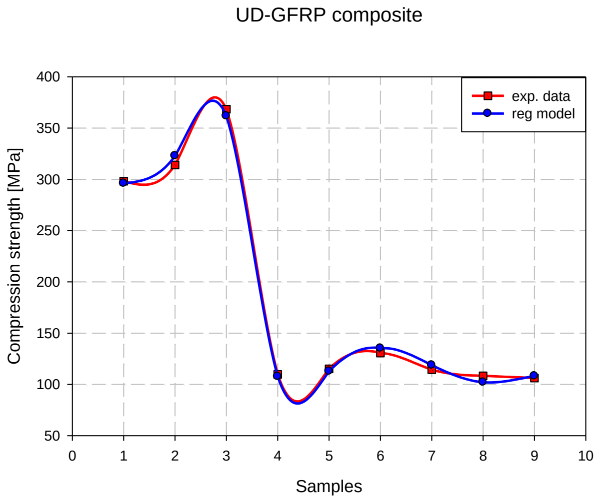


Fig. 4 Graphical representation of experimental data and regression model

4 CONCLUSIONS

This study focused on the dependence between reinforcement grades, fibres orientation angle and compression strength of the UD-GFRP composite. The research was aimed at the comparative assessment between experimental tests at compression strength and a regression analysis of the active experiment.

The compression strength of UD-GFRP composites structure made from 16 layers has been examined. The used materials were UD glass fibres 225 g/m^2 and polyester resin Lerpol TIX 3603/R.

The data obtained by mechanical tests, of the unidirectional fiber composite plates, have been used to develop mathematical models regarding the influence of the structure on the compression strength of the UD composite.

Regression analysis was used for the investigation of relationships between involved variables, method that generated mathematical models.

Mathematical regression model have been successfully developed to predict theoretical compression strength of the composite.

We can conclude that the results have excellent agreement with measured values, they shown high accuracy.

The *R* square value obtained for the regression model was 0.9976, which is in the well acceptable range and confirms the validity of the model.

5 REFERENCES

►Eswari, S., Raghunath, P.N., Kothandaraman, S. (2011). Regression modeling for strength and toughness evaluation of hybrid fibre reinforced concrete, ARPN Journal of Engineering and Applied Sciences, Vol 6 (5), pp. 1-8.

►Jelf, P.M., Fleck, N.A. (1992). Compression failure mechanisms in unidirectional composites, Journal of Composite Materials, Vol. 26 (18), pp. 2706-2726.

►Lin, J.T., Bhattacharyya, D., Kecman, V. (2003). Multiple regression and neural networks analyses incomposites machining, Composites Science and Technology, Vol. 63, pp. 539–548.

►Lomov, S.V., Verpoest, I. (2000). Compression of woven reinforcements: a mathematical model, Journal of Reinforced Plastics and Composites, Vol. 19 (16), pp. 1329-1350.

►Mavromatidis, E.L., Bykalyuk, A., Lequay, H. (2013). Development of polynomial regression models for composite dynamic envelopes' thermal performance forecasting, Applied Energy, Vol. 104, pp. 379-391.

►Ostertagová, E. (2012). Modelling using polynomial regression, Procedia Engineering, Vol. 48, pp. 500-506.

►Palanikumar, K., Davim, PJ. (2007). Mathematical model to predict tool wear on the machining of glass fibre reinforced plastic composites, Materials & Design, Vol. 28 (7), pp. 2008-2014.

►Palanikumar, K., Karunamoorthy, L., Manoharan, N. (2006). Mathematical model to predict the surface roughness on the machining of glass fiber reinforced polymer composites, Journal of Reinforced Plastics and Composites, Vol. 25 (4), pp. 407-418.

►Sabău, E., Bâlc, N., Bere, P., Borzan, C.S., Ceclan, V. (2013). Experimental study on mechanical behavior of glass fiber reinforced polymer composites under axial compression, Academic Journal of Manufacturing Engineering - AJME, Vol. 11 (3), pp. 110-113.

►Sabău, E., Iancău, H., Potra, T., Hancu, L., Popescu, A. (2009). Cubic spline of regression with application at composite materials, MSE 2009, The 4th International Conference on Manufacturing Science and Education, pp. 71-74.

►Satyanarayana, G., Naidu, S.G., Babu, H.N. (2018). Artificial neural network and regression modelling to study the effect of reinforcement and deformation on volumetric wear of red mud nano particle reinforced aluminium matrix composites synthesized by stir casting, Boletín de la Sociedad Española de Cerámica y Vidrio, Vol. 57 (3), pp. 91-100.

►Uyanık, K.G., Güler, N. (2013). A study on multiple linear regression analysis, Procedia - Social and Behavioral Sciences, Vol. 106, pp. 234-240.