RESEARCHES CONCERNING THE MODIFICATION OF TEETH PROFILE OF CYLINDRICAL GEARS IN FRONTAL PLANE LOADED BY LINEARLY FORCES

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Abstract : *The paper presents the recent results of research on changing the profile of the teeth of cylindrical gears in the frontal plane in the case of loading with linearly variable forces. The finite element method is used to determine the elastic deformations of the cylindrical gear teeth considering the errors of execution, manufacture and operation of the gear through the loading cases with linearly variable forces on the width of the gear tooth.The load with linearly variable forces on the width of the gear tooth allows the most favorable simulation of the gear errors.*
Key words*: tooth top correction, finite element, shape modification.*

1. GENERAL CONSIDERATIONS

The gear tooth profile modified within the front plane by tooth top correction and/or tooth base, gets a more and more utilization, mainly to the high loaded gearing from the planes, *cars*, boats, gas turbines, etc. The main reason, which imposes the using of the profile modifications, is the possibility to compensate both the elastic deformation of the gearing loaded teeth and the manufacturing and assembly errors.

In order to perform this tooth profile modification, the STAS 821-82 standard specifies the reference rack modification and establishes the limits for the top tooth flanking. These recommendations are made only with respect to the modulus, and do not take into account the actual gearing conditions.

The value of the profile depth modification must be considered in such a way that it should eliminate the edge collisions, which might appear due to the step errors and to the loaded teeth elastic deformations.

The value of the depth profile modification is depending on the accuracy degree of the gear manufacturing and teeth elasticity.

2. THE PRINCIPLE OF THE METHOD

The profile modifications are known generally as top tooth corrections having the Δ_a value (fig.1). Suddenly load changes into the getting in and out gearing, generates vibrations and noise. The same inconveniences are caused into the points of passing from be-even gearing to uni-even gearing. Top tooth profile correction must de designed in such a way that the local load should increase as uniform as possible and it should gradually decrease to zero, during unloading.

The depth of the profile modification on the teeth top, it is recommended (Suhoenkov, 1992), based on the above mentioned observations, mainly using the high precision gears, to be calculated with the formula:

$$\Delta_a = f_{\Sigma pb} + \delta_e \qquad (1)$$

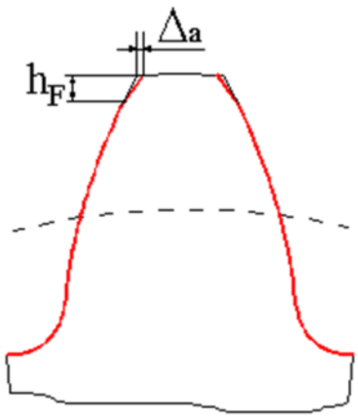

Fig. 1. **Tooth with modified profile.**

Where:

- Δa represents the profile modification depth, at the tooth top
- δ_e is the elastic deformations size of the loaded gear teeth

- $f_{\Sigma pb}$ is the square average deviation of the pinion and gear step maximum deviations and can be calculated with formula:

$$f_{\Sigma pb} = \sqrt{f_{pb1}^2 + f_{pb2}^2} \qquad (2)$$

It can be concluded that: the profile modification value is influenced by the manufacturing precision class that the gear has been machined in (step and profile accuracy), on one side, and by the quality of the material used for gear production, by its elastic deformation.

When choosing the profile modification parameters value, it is recommended to take into account different situations.

We recommend (Gyenge, C., Mera, M. 1996) that the Δ_a depth profile modification, to be equal to the maximum elastic deformation during gearing, in the case of precise and very precise gears (IT5-6 and IT3-4), calculated using the presented methodology, and to add the square average deviation of the maximum deviations of the steps of the gears, caused mainly by the manufacturing errors; Δ_a to be calculated with formula (1) and for the gears corresponding to the IT 7-8 precision class, mainly the step errors should be taken into account in Δ_a calculation.

3. HOW TO CALCULATE THE Δ_a CORRECTION DEPTH VALUE FOR THE OUTER CYLINDRICAL GEAR TEETH, USING THE FINITE ELEMENT METHOD

The correct calculation of the correction parameter value for the tooth top profile is very important, by its influence on the good function of the gearing.

Within the undertaken researches the modification depth of the gear tooth profile has been considered as depending on the elastic deformation of loaded gearing and of its manufacturing errors, mainly by the acceptable step errors of pinion and gear.

Elastic deformation, which appear during loaded gearing, are calculated by finite element analyses, for every particularly gearing. Manufacturing errors, mainly those of maximum stem deviations for the pinion and the gear, were taken from standards [2], based on the acceptable deviations.

Let us consider a gear, fixed on a shaft. The 3D model (fig. 3.) developed in this way, has been analyzed using the ALGOR finite element method.

Fig. 2. The shaft-gear 3D model.

The cylindrical gear taken for study has z=30 teeth and m = 4 modulus. The normal force per tooth is 1500 N and it is linear distributed on its width. For the chosen alternative, we considered the 6-th accuracy class, after smooth functioning criteria, the type C of fitting, minimal clearance fitting between flanks c, acceptable step error of the pinion and gear being +/- 12 μm.

In the case when a tooth direction error exists, due to the manufacturing, or to the elastic deformation appeared during functioning, or to assembly errors, or to the simultaneous existence of these errors, the loading is done by a linear distribution (Mera et al., 1995).

4. LOADED BY LINEAR FORCES

Three cases have been considered: $b=b_{cal}$, $b_{cal}<b$ and $b_{cal}>b$, where b_{cal} represents the width taken into account for calculations (fig. 4, 5, 6).

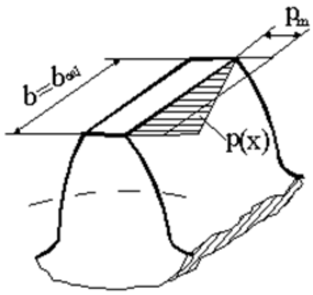

Fig. 3. **The forces linear distribution, for the case $b=b_{cal}$**

A linear function is considered, such as: p(x)=mx+n, which describes the force linear

evolution ob the b width of the gear tooth. By imposing the limit conditions:

$$p(0) = p_{\max} \Rightarrow n = p_{\max}$$
$$p(b) = 0 \Rightarrow mb + n = 0 \Rightarrow m = -\frac{p_{\max}}{b} \qquad (3)$$

That leads to:

$$p(x) = p_{\max}(1 - \frac{x}{b}) \qquad (4)$$

The F force, which is applied on the tooth, can be calculated as being:

$$F = \int_0^b p(x)dx = \int_0^b p_{max}(1-\frac{x}{b})dx = p_{max}\frac{b}{2} \qquad (5)$$

On the b width of the tooth, a number (n) of knocks is established, so (n-1) subintervals are obtained. The force corresponding to each subinterval can be calculated:

$$F_{int} = \frac{2F}{b}(x_1 - x_0)(1-\frac{x_1+x_0}{2b}) \qquad (6)$$

The following recurrence formula was established:

$$F_k = \frac{F\{2(n-1)-[k+(k-1)]\}}{(n-1)^2} \qquad (7)$$

Where: k represents the subinterval numbers

F_k represents the linear variable force on k subinterval

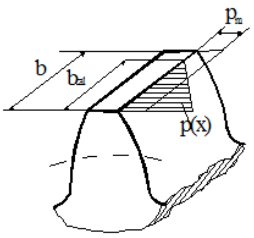

Fig. 4. **The forces linear distribution, for the case $b_{cal}<b$**

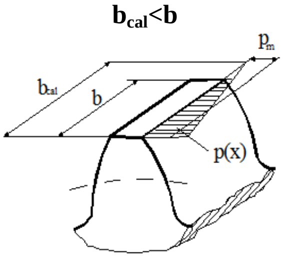

Fig. 5. **The forces linear distribution, for the case $b_{cal}>b$**

If we consider the case when $b>b_{cal}$ and $b<b_{cal}$ respectively, the linear variable force on every subinterval is calculated as follows:

$$F_k = \frac{F\{2i-[k+(k-1)]\}}{i^2} \qquad (8)$$

Where: $b_{cal} = ib / (n-1)$

i represents the number of suinteervals on the tooth width.

The table 1 presents the linear distributed forces, calculated with the above presented formulas, applied into the knocks of the finite element network, within the FEM analyses performed for the considered cases.

Table 1 The linear distributed forces value, for cases $b=b_{cal}$, $b>b_{cal}$ and $b<b_{cal}$.

Numărul nodului din rețeaua de elemente finite în care se aplică forța	Valoarea forțelor liniar distribuite în nodurile rețelei [N], pentru cazurile:		
	$b = b_{cal}$	$b > b_{cal}$	$b < b_{cal}$
	Numărul subintervalelor de calcul, *i*		
	$i = 6$	$i = 5$	$i = 7$
1	229,165	270	199
2	416,665	480	367,34
3	333,33	360	306,12
4	249,995	240	244,9
5	166,665	120	183
6	83,33	30	122,45
7	20,83		61,22
8			15,3

5. NUMERICAL RESEARCHES

The maximal value of the tooth elastic deformation, for the case $b=b_{cal}$, is at the end of the gear, closer to the torsion moment loading and it has 5 μm value (fig. 7).

The depth value of the modified profile is for this case: Δ_a = 20 μm, corresponding to a 5 μm tooth elastic deformation and to a 12 μm step error.

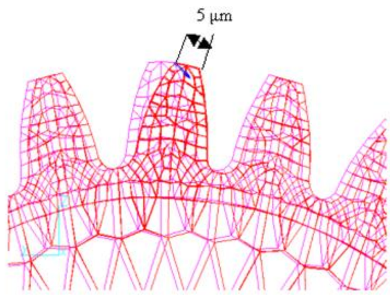

Fig. 6. **The initial tooth state, overlap on the elastic deformed state modified profile gear tooth.**

The STAS 821/82 standard recommends, for this case, the flanking coefficient for the reference rack tooth top to be Δ^*_{aF} = 0.008, which means that the flanking depth is Δ_{aF} =0,032 mm for a m = 4 mm modulus.

In the case $b_{cal}<b$, when the gearing is less loaded and/or the tooth direction total value is higher, the maximal elastic deformation of the gear teeth is 5,66 μm, on the end of the gear closer to the side where the torsion moment is being applied. The profile correction depth value, calculated with formula (2), is 22,6 μm.

For the case $b_{cal}>b$, when the gear is strongly loaded and/or the total tooth direction deviation has a smaller value, the tooth maximal elastic deformation is on the end closer to the zone where the torsion moment is applied, and has the 4,411 μm value. The profile top tooth correction depth value is 21,4 μm.

The gear teeth elastic deformations, in the case of linear distributed loads, are the smallest for the case $b_{cal}>b$, when the gear tooth direction errors, or manufacturing errors, or the elastic deformations during functioning, or assembly errors, are smaller, and/or the gearing is strongly loaded.

6. PERSPECTIVES

Future researches will be orientated to perform studies which should lead to establish the following things:

- the influence of the tooth width load distribution on parameters value for gear teeth profile modification
- the influence of the gear with respect to the roller bearings, on parameters value for gear teeth profile modification
- the influence of the manufacturing and assembly errors on parameters value for gear teeth profile modification.

7. REFERENCES

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