# MULTI DEGREE OF FREEDOM ROBOTIC ARM ANALYSIS VIA NUMERICAL APPROACH FOR SMOOTH JOINT MOTION PLANNING

**Marjan DJIDROV[1*], Anastasija IGNJATOSKA[1]**
*[1]Ss. Cyril and Methodius University in Skopje, Faculty of Mechanical Engineering*
**Corresponding author Marjan Djidrov: marjan.djidrov@mf.edu.mk*

***ABSTRACT:*** *The significance of kinematics and dynamics in multi-degree of freedom systems is evident in robotics, where they play crucial roles in achieving precise positioning and orientation. These concepts are essential for various aspects of robotic functionality, including motion planning, manipulation, control, performance optimization, and ensuring safety and reliability. Therefore, creating a mathematical model that represents how the multi-degree-of-freedom system moves based on its joints and links is essential for understanding the robotic manipulator's behavior. In this paper a kinematic model of a six degree-of-freedom robotic manipulator is presented. The analysis of the manipulator includes the determination and tracking of the values of the kinematic characteristics for gaining insights into the robot's motion capabilities while considering its limitations and to determine its behavior. The goal is to enabling effective motion planning in different cases that's considerate both a straight and a curve paths. Smooth transitions were achieved between joint positions which enhanced the stability and accuracy of the end-effector.*

***KEYWORDS****: Manipulator, Robotic arm, Motion planning, Jacobians, Kinematics*

## 1 INTRODUCTION

The versatility and adaptability of robot manipulators make them valuable assets across various industries where tasks require heavy lifting, precision, and reach. They are machines equipped with joints, sensors, and end-effectors, and are programmed to perform specific actions, like welding, painting, cutting, material handling (Singh & al, 2013). Kinematic identification and calibration (Kana, S. & al, 2022) are essential for ensuring that industrial robots operate reliably, accurately, and safely, ultimately contributing to improved product quality, efficiency, and cost-effectiveness in manufacturing processes. Therefore, creating a mathematical model that represents how the robot moves based on its joints and links is essential for understanding the robot's behavior. However, to improve the accuracy and performance of robotic systems, especially in applications where precise positioning is essential, measurements of the robot poses are needed (Boby, R. A., & Klimchik, A, 2021). Furthermore, the kinematic parameters of the robot's model are estimated or adjusted to minimize the difference between the predicted and actual robot poses. This means fine-tunes the model to better match the real-world behavior of the robot. Once the kinematic parameters are accurately estimated, any remaining errors between the predicted and actual poses can be compensated for.

The importance of robot kinematics in achieving precise positioning and orientation is evident across various aspects of robotic systems, including motion planning, manipulation and control, performance optimization, and safety and reliability. An understanding of kinematics is fundamental to the design, operation, and advancement of robotic technology. Through kinematic analysis and optimization, engineers can minimize the time and energy required (Soori & al, 2023) for the robot to move between different positions, leading to improved productivity. Furthermore, by precisely controlling the robot's joint motions for consistent performance of repetitive tasks, the variations in task execution can be minimized that leads towards greater consistency in product quality. Kinematics is fundamental for coordinating the motions of multiple robots or robotic systems within an automated manufacturing environment (Gogouvitis, X. V., & Vosniakos, G. C., 2015). By understanding the kinematic relationships between different robotic components, engineers can synchronize their motions to achieve seamless operation. Robot kinematics is central to achieving precise positioning (Wu, J. & al, 2019), (Gadringer & al, 2020), efficient motion (Yonezawa & al, 2024), and safe operation of industrial robots across a wide range of applications (Messay & al, 2016). It provides the foundation to design and control robotic systems that meet the demanding requirements of advanced manufacturing.

The robotic manipulator with six degree-of-freedom (DOF) is presented in Figure 1. Joint 1 allows the manipulator to rotate around the base, while Joint 2 and 3 moves up and down allowing it to reach forward and backward and providing vertical movement. Joint 4 rotates the upper arm, Joint 5 tilts the wrist up and down, and Joint 6 spins the end-effector around its axis. Each joint can rotate within a specific range of angles, known as joint limits and all joints can be adjusted to specific angles or positions. By changing the angles of the joints, the overall configuration of the manipulator changes, affecting the position and orientation of its end-effector (Sciavicco, L., & Siciliano, B. 2012). The stiffness of a robot's structure can vary as its posture or configuration changes. This variation can lead to nonlinearities in the distribution of stiffness throughout the robot's workspace (Wu, K. & al, 2022). As a result, achieving precise machining or manipulation becomes challenging because the robot's behavior may not be predictable or consistent across its entire range of motion.

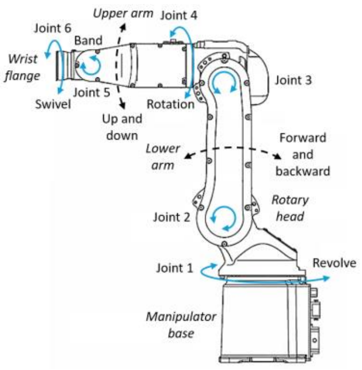


**Fig. 1 6 DOF industrial robot**

In the realm of robotics, the forward kinematics problem involves establishing the end-effector's pose by considering the provided joint variables. Conversely, the inverse kinematics problem involves determining the joint configurations or angles of a robotic manipulator required to reach a desired position and orientation for its end-effector. Solving the inverse kinematics problem is essential in robotics, however, because of the complexity of the problem (Kucuk, S., & Bingul, Z., 2006), there isn't a single universal solution applicable to all robotic systems. Instead, various approaches can be employed, each with its own advantages and limitations. Numerical methods use iterative techniques to approximate solutions (Aristidou, A., & al, 2018). They tend to have higher computational costs, longer execution times, and may encounter issues such as local minima and numerical errors. Closed-form methods provide solutions in explicit mathematical forms, often based on the geometry of the robotic manipulator (Zaplana, I. & al, 2022). They have advantages such as lower computational cost and faster execution time compared to numerical methods. However, they may not be applicable to all types of manipulators and end-effector poses. These methods include strategies based on matrix manipulations, arm angle parameter definitions, and geometric methods (Kim, J. S., & al, 2015) or artificial neural networks (Cagigas-Muñiz, D., 2023).

In the next sections, we delve into different aspects of the multi-degree-of-freedom system. Section 2 focuses on kinematics and the methods used for numerical analysis. Section 3 introduces a specific 6 DOF serial robotic arm's kinematic model and discusses the challenges associated with trajectories. Our research findings and their analysis are presented in Section 4, followed by Section 5, which summarizes our conclusions.

## 2 KINEMATIC ANALYSIS OF AN INDUSTRIAL SERIAL MANIPULATOR

For modeling robotic manipulators, the Denavit-Hartenberg (DH) method provides a systematic way to describe the geometry and kinematics of a manipulator. Frames are assigned to each joint of the manipulator, starting from the base frame and progressing towards the end-effector frame. The DH parameters used in this method, as shown in Figure 2 includes, $\theta i$ as joint angle and $\alpha i$ as angle of rotation, di is the length of the link and ai as distance between axes. These DH parameters are essential for defining the transformation between adjacent frames in the manipulator. By appropriately choosing and assigning these parameters, the kinematics of the manipulator can be accurately represented, allowing for control and trajectory planning (Benotsmane, & al, 2021). In DH parameterization, each joint of the robotic manipulator is assigned a sequential number starting from 1 to n, where 1 represents the first joint nearest to the base and n represents the last joint of the manipulator, which is typically located at the end-effector.

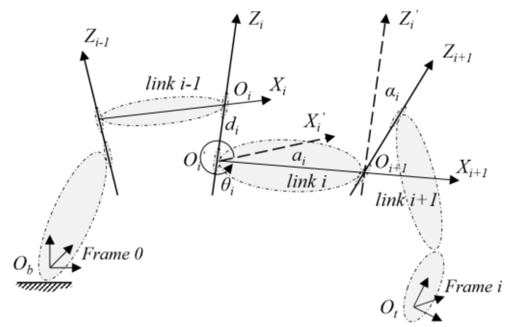


**Fig. 2 Schematic of robot links and DH parameters**

Forward kinematics as one of the fundamental concepts in robotics deals with the determination of the position and the orientation of the end-effector, the tool or end point of a robotic arm given the joint variables, such as angles of its individual joints and the link length. To achieve forward kinematics, one typically defines a series of homogeneous transformation matrices for each joint of the robot. These matrices describe the transformation from one coordinate system to another as the robot moves through its various joint configurations. By combining these transformations, the position and orientation of the end-effector relative to a fixed reference frame can be calculated. The position and orientation of the tool frame in relation to the base frame are determined by combining the transformations (both translation and rotation) between each intermediate frame and the base frame using homogeneous transformation matrices. Each intermediate frame provides information about how much the robot has translated and rotated from the base frame. By combining these transformations using homogeneous transformation matrices, we can accurately determine the position and alignment of the tool frame relative to the base frame. These matrices are typically 4x4 matrices $T_i^{i-1}$ $(i = 1,..6)$:

$$T_i^{i-1} = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & a_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

The process of calculating the position and orientation of the tool frame in relation to the base frame involves multiplying the homogeneous transformation matrices of each intermediate frame with respect to the base frame. This multiplication effectively combines the translation and rotation information from each frame to determine the overall position and orientation of the tool frame.

$$T_6^0 = T_1^0\, T_2^1\, T_3^2\, T_4^3\, T_5^4\, T_6^5 \tag{2}$$

$$T_6^0 = \left[\begin{array}{c|c} Rotation\ matrix\ R\ (3\ x\ 3) & Translation\ vector\ T\ (3\ x\ 1) \\ \hline Perspective\ transformation & Scaling \end{array}\right] \tag{3}$$

The submatrix R represents the rotation, while T is the translation part of the homogeneous transformation matrices. The rotation matrix R (3 x 3) is formed by the nine elements in three columns with notation r for describing rotations in three-dimensional space. The translation vector T (3 x 1) is represented by three elements in a column, with notation t describing how much the coordinate frame has been moved in each of the x, y, and z directions. It follows:

$$T_6^0 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{22} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

## 2.1 Inverse kinematics

Finding the position and orientation of the end-effector given the joint angles of the robot, i.e. calculating how the robot's joints move the end-effector in space is a task for forward kinematics. On the other hand, finding the joint angles required to place the end-effector at a specific position and orientation in space is a task related to inverse kinematics. Robots typically operate in joint space, where movements are defined by the angles of the robot's joints. However, tasks are often specified in Cartesian space, where positions and orientations are described in terms of coordinates and orientation matrices. Converting from Cartesian space to joint space involves solving the inverse kinematics problem. This requires finding the joint angles that achieve the desired end-effector position and orientation (Kucuk, S., & Bingul, Z., 2006). The general problem of inverse kinematics can be stated via the desired position and orientation of the end-effector $T_d$ and 4 × 4 homogeneous transformations (Spong, & al, 2020), namely, to find (one or all) solutions of the equation:

$$T_n^0(\theta_1, \dots \theta_n) = T_d \tag{5}$$

Among the most challenging issues in robotics is inverse kinematics. The task is to find the values for the joint variables $\theta_1, \dots \theta_n$ that satisfied the equation. Because each link in the robotic manipulator has a transformation matrix that describes how it moves relative to the previous link or the robot's base, by taking the inverses of these transformation matrices and premultiplying them (Zhang, L., & al, 2015), it can be combine the effects of each link's movement to find the joint angles required to achieve the desired end-effector pose. Consequently, for:

$$[T_1^0(\theta_1)]^{-1}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{22} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = [T_1^0]^{-1}\, T_1^0\, T_2^1\, T_3^2\, T_4^3\, T_5^4\, T_6^5 \tag{6}$$

$$\begin{bmatrix} c_1 & s_1 & 0 & 0 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{22} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = T_2^1\, T_3^2\, T_4^3\, T_5^4\, T_6^5 \tag{7}$$

it follows:

$$\theta_1 = atan2(t_y, t_x) - atan2(-s_1 t_x + c_1 t_y, \pm\sqrt{t_x^2 + t_y^2 + (-s_1 t_x + c_1 t_y)^2}) \tag{8}$$

$$\theta_3 = atan2(a_3, d_4) - atan2(K, \pm\sqrt{a_3^2 + d_4^2 - K^2}) \tag{9}$$

where, simplify notations are $c_i$ for $cos(\theta_i)$, and $s_i$ for $sin(\theta_i)$, and

$$K = [t_x^2 + t_y^2 + t_x^2 - a_2^2 - a_3^2 - (-s_1 t_x + c_1 t_y)^2 - d_4^2] / 2a_2 \tag{10}$$

Taking into consideration that:

$$[T_3^0(\theta_2)]^{-1}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{22} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = [T_1^0\, T_2^1\, T_3^2]^{-1}\, T_1^0\, T_2^1\, T_3^2\, T_4^3\, T_5^4\, T_6^5 \tag{11}$$

$$\begin{bmatrix} c_1 c_{23} & s_1 c_{23} & -s_{23} & -a_2 c_3 \\ -c_1 s_{23} & -s_1 s_{23} & -c_{23} & a_2 s_3 \\ -s_1 & c_1 & 0 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{22} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = T_4^3(\theta_4)\, T_5^4(\theta_5)\, T_6^5(\theta_6) \tag{12}$$

It follows:

$$\theta_2 = atan2[(-a_3 - a_2 c_3)t_z - (c_1 t_x + s_1 t_y)(d_4 - a_2 s_3), (a_2 s_3 - d_4)t_z - (a_3 + a_2 c_3)(c_1 t_x + s_1 t_y)] - \theta_3 \tag{13}$$

$$\theta_4 = atan2(-r_{13} s_1 + r_{23} c_1, -r_{13} c_1 c_{23} - r_{23} s_1 c_{23} + r_{33} s_{23}). \tag{14}$$

where, simplify notations are $c_{ij}$ for $cos(\theta_i + \theta_j)$, and $s_{ij}$ for $sin(\theta_i + \theta_j)$. In a singular configuration (Doan, N. C. N., & Lin, W., 2017), specifically if $\theta_5 = 0$, the joint axes 4 and 6 line up and cause the same motion of the last link of the robot. Furthermore, considering that:

$$[T_4^0(\theta_4)]^{-1}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{22} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = [T_1^0\, T_2^1\, T_3^2\, T_4^3]^{-1}\, T_1^0\, T_2^1\, T_3^2\, T_4^3\, T_5^4\, T_6^5 \tag{15}$$

$$\begin{bmatrix} c_1 c_{23} c_4 + s_1 s_4 & s_1 c_{23} c_4 - c_1 s_4 & -s_{23} c_4 & -a_2 c_3 c_4 + d_3 s_4 - a_3 c_4 \\ -c_1 c_{23} s_4 + s_1 c_4 & -s_1 c_{23} s_4 - c_1 c_4 & s_{23} s_4 & a_2 c_3 s_4 + d_3 c_4 + a_3 s_4 \\ -c_1 s_{23} & -s_1 s_{23} & -c_{23} & a_2 s_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{22} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = T_5^4(\theta_5)\, T_6^5(\theta_6) \tag{16}$$

It follows:

$$\theta_5 = atan2[-r_{13}(c_1 c_{23} c_4 + s_1 s_4) - r_{23}(s_1 c_{23} c_4 - c_1 s_4) + r_{33}(s_{23} c_4), r_{13}(-c_1 s_{23}) + r_{23}(-s_1 s_{23}) + r_{33}(-c_{23})] \tag{17}$$

At last, since:

$$[T_5^0]^{-1}\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{22} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = [T_1^0\, T_2^1\, T_3^2\, T_4^3]^{-1}\, T_1^0\, T_2^1\, T_3^2\, T_4^3\, T_5^4\, T_6^5 = T_6^5(\theta_6) \quad (18)$$

The following angle is obtained:

$$\theta_6 = atan2[-r_{11}(c_1c_{23}s_4 + s_1c_4) - r_{21}(s_1c_{23}s_4 + c_1c_4) + r_{31}(s_{23}s_4), r_{11}[(c_1c_{23}c_4 + s_1s_4)c_5 - c_1s_{23}s_5] + r_{21}[(s_1c_{23}c_4 - c_1s_4)c_5 - s_1s_{23}s_5] - r_{31}(s_{23}c_4c_5 + c_{23}s_5)] \quad (19)$$

In this manner, solving the inverse kinematics of the 6 DOF manipulator needs addressing twelve sets of nonlinear equations. The primary unknown is $\theta_1$ that appears on the left side of the equation (6). Furthermore, the twelve nonlinear matrix elements on the right side of the equation can be either zero, constant, or functions of $\theta_2$ through $\theta_6$. Therefore, by equating the elements on both sides of the equation, the joint variable $\theta_1$ is solved as functions of $r_{11}, r_{12}, \ldots, r_{33}, t_x, t_y, t_z$, and fixed link parameters. Once $\theta_1$ is determined, subsequently the remaining joint variables can be solved using this procedure.

## 2.2 Approximations via Jacobians

In the context of robot kinematics, which deals with the motion of robots without considering the forces that cause the motion, the Jacobian matrix is used to relate the velocities of the robot's joints to the velocity of the end-effector (Donelan, P., 2010). The Jacobian helps to understand how changes in joint angles affect the velocity of the end-effector (Corke, P., 2023). While forward kinematics tells where the end-effector is located, however it doesn't tell us how quickly it's moving or in what direction. Namely, information about the relationship between joint velocities and end-effector velocities is not obtained. The Jacobian matrix bridges this gap by providing a linear approximation of the relationship between joint velocities and end-effector velocities near a specific configuration of the manipulator.

$$\frac{d}{dx}x(t) = J\dot{\theta} \quad (20)$$

The Jacobian matrix can help in determining how much each joint angle needs to change to bring the end-effector closer to the desired position. Therefore, by iteratively adjusting the joint angles based on the difference between the current end-effector position and the desired position, according to the relationship described by the Jacobian matrix, it can converge towards the desired end-effector position and orientation. However, the complexities involved in utilizing the Jacobian matrix in robotics, including issues related to parameter availability, system stability, and control design (Yu, W., & Perrusquía, A., 2020). To compute the change in joint angles $\Delta\theta$ needed to move from the current position to the desired position, the relation of the error vector $\Delta E$ between the desired end-effector position and the current end-effector position and the inverse of the Jacobian matrix, can be use:

$$\Delta\theta = J^{-1}(\theta)\Delta E \quad (21)$$

The Jacobian matrix typically has two distinct components, the first part corresponds to linear velocities and the second part corresponds to angular velocities:

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} \quad (22)$$

Incorporating the rotational matrix $R_i$, the displacement vector $d_{i-1}$, and the displacement vector $d_n$, the Jacobian matrix can be written as:

$$J = \begin{bmatrix} R_{i-1}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \text{x}\,(d_n - d_{i-1}) \\ R_{i-1}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \end{bmatrix} \quad (23)$$

Since, each joint of the robotic manipulator has minimum and maximum limits of motion, the objective is to reduce the error between the desired position and orientation of the end-effector and the actual position and orientation calculated by the robotic manipulator's kinematics. Consequently, via iterative process a desired end-effector position and orientation can be achieved. It begins with the initialization step, where initial joint angles, desired end-effector position and orientation, convergence criteria, or maximum number of iterations are specified. Based on selected joint angles, via using forward kinematics the current end-effector pose can be determined. It is followed by the step for computing pseudo-inverse, i.e. Jacobian inverse. Then the error vector can be computed by determining the difference between the desired end-effector pose and the current end-effector pose using forward kinematics. The next step involves computation of the joint angle change, which means multiplying the pseudo-inverse of the Jacobian by the error vector. After updating the current joint angles, next is to check if the error is below a predefined threshold or if the change in joint angles is negligible. If convergence criteria are satisfied, the output of the final joint angles that achieve the desired end-effector pose can be generated. However, if not, can proceed to the next iteration and increment the iteration counter until convergence requirements are fulfilled.

## 3 KINEMATIC MODEL OF A 6 DOF ROBOTIC MANIPULATOR

In robotic systems, various components, including joints, links, and the end-effector, are spatially characterized using coordinate systems. Each joint in a robotic manipulator typically possesses its own dedicated coordinate system. This localized system serves to define the movement of the joint concerning a fixed reference point, such as the robot's base. These joint-specific coordinate systems pivot with their corresponding joints, providing a local frame of reference for analyzing the motion and positioning of each individual joint and its associated link. A representation of a robotic manipulator, with coordinate systems assigned to each joint using the DH method is presented in Figure 3.

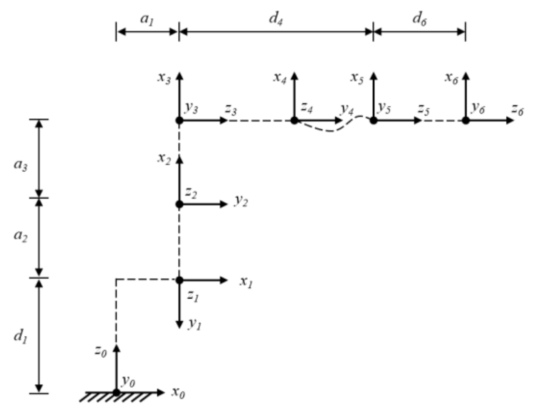

**Fig. 3 Kinematic model of a 6 DOF robotic manipulator**

The utilized manipulator operates within specific joint limits to ensure precision and safety. Each axis has defined ranges: -60 to 60 degrees for the first, 0 to 90 degrees for the second, -80 to 80 degrees for the third, -180 to 180 degrees for the fourth, and -80 to 80 degrees for the fifth. The sixth axis offers a wide range of motion from -270 to 270 degrees. The DH parameters for a 6 DOF robotic manipulator are:
$\{\theta_i, \alpha_i, a_i, d_i\}$, where $i$ ranges from 1 to 6, with values as follows: $\{-\pi/2,\ 0, -\pi/2,\ \pi/2, -\pi/2, 0\}$ for $\alpha_i$, $\{160, 580, 125, 0, 0, 0\}$ for $a_i$[mm], and $\{430, 0, 0, 240, 0, 410\}$ for $d_i$[mm].

The analysis of the serial robotic manipulator includes the determination and tracking of the values of the kinematic characteristics related to angles of each of the six joints, the angular velocities or accelerations. Determining possible unexpected changes of these parameters, out of determinate limits for given task, in any of the six joints of a 6 DOF robotic manipulator is crucial for various reasons. Firstly, it enhances safety by detecting rapid changes in joint velocities that could result in hazardous movements or unexpected accelerations, posing a risk to nearby operator or equipment. Secondly, it ensures the smooth and controlled operation of the robot, especially during complex tasks where precise motion control is essential. Thirdly, by determining and monitoring these parameters, it enables the detection of deviations from expected performance to ensure continuous production flow. Additionally, optimize the robot's performance, improving efficiency and reducing cycle times in manufacturing processes. Moreover, it contributes to quality control by ensuring consistent motion profiles, which is particularly important in applications requiring high accuracy, such as machining or inspection.

## 4 RESULTS AND ANALYSIS

To gain insights into the robot's motion capabilities while considering its limitations and to determine its behavior, with goal to enabling effective motion planning, or additional control and optimization, in the following are presented the results related to kinematic parameters. The case that is considerate includes moving in a straight line, which is the diameter of a given circle d=250mm. And then, the movement continues along a curve, which refers to a part of the circle. That is, it is necessary to move along a path in the form of a semicircle, which contains both a straight and a curve line.

In Figure 4, changes of the manipulator joint angles are presented, where joints 3, 4, and 6 undergo larger angular displacements compared to joints 1, 2, and 5. When t=355, a noticeable change in values is observed, which refers to the fact that a change also occurs in the path. Specifically, when the robot manipulator moves along the part of the trajectory that is a straight line and passes into the part when it follows a curve, the part of a semicircle. Furthermore, it can be noticed that the joint angles are within limits and there are no sudden changes. This suggests that the manipulator is operating safely, smoothly, and predictably, complying with its constraints and performing optimally. The manipulator executes its tasks as intended without abrupt or jerky movements, while operating within its physical and mechanical constraints. For tasks that require precision, smooth operation is essential. Additionally, this ensures that the robotic manipulator can operate optimally, maximizing efficiency and productivity. Through further consistent and controlled motion, will allow better planning and execution of given tasks, ultimately leading to improved overall performance.

Therefore, realizing which joints experience larger angular displacements is crucial for both the design of the robot's mechanical structure and the development of motion planning algorithms for controlling its movement.

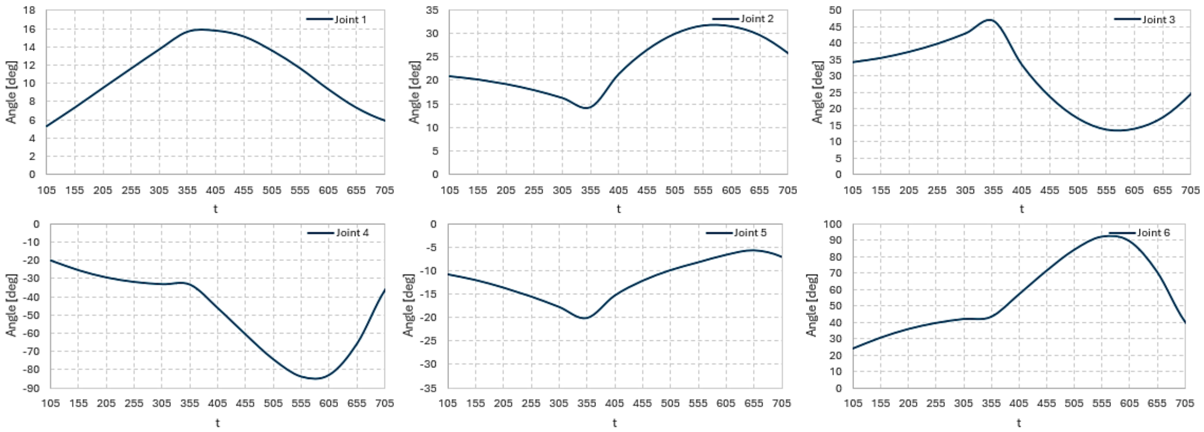


**Fig. 4 Robotic manipulator joint angular displacement**

When changes in joint angles occur smoothly and remain within predetermined limits, it indicates that the corresponding joint velocities, as shown in Figure 5, are similarly changing smoothly and consistently. This consistency in motion ensures that the manipulator moves predictably and maintains stability throughout the given tool path. By ensuring that the transitions between configurations are smooth and gradual, the manipulator's components experience less sudden stress and wear. This can lead to longer lifetimes for the robot and its parts, reduced maintenance requirements, and overall improved reliability. Additionally, excessive velocities can lead to instability and overshooting. Therefore, controlling joint velocities is essential for the safe and reliable operation of the robotic manipulator. Overall, smooth and consistent changes in both angles and velocities result in smoother, more natural-looking motion, which is particularly important for applications requiring high precision and delicate workpieces.

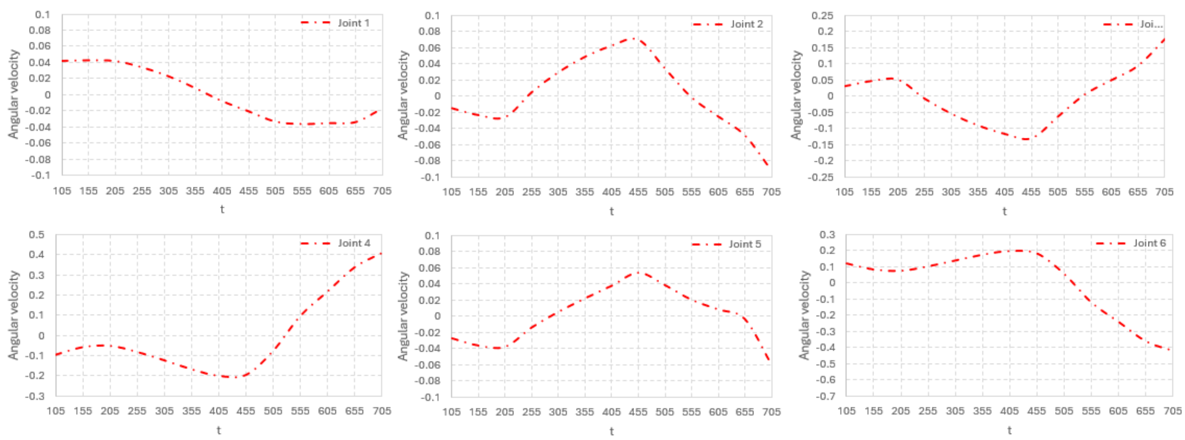


**Fig. 5 Robotic manipulator joint angular velocities**

## 5 CONCLUSION

In a robotic manipulator, which consists of multiple segments connected by joints, each joint allows movement along one or more axes. Respectively, joints of the manipulator can be rotated to different angles, allowing the end-effector to reach various positions and orientations in the workspace, thereby facilitating various tasks and applications. By controlling the angles of the joints, the robot can manipulate objects or perform tasks in three-dimensional space with precision and flexibility. Smooth transitions between joint positions enhance the stability and accuracy of the end-effector, making it easier to perform given tasks. For operations involving delicate materials and precision movements, smooth motion is crucial to avoid damage or errors. Additionally, gradual movements reduce mechanical vibrations, which can otherwise affect the precision of the end-effector. In environments where robots work alongside humans, smooth and predictable movements reduce the risk of accidents and enhance safety. However, by using the known joint angles of a robotic arm and applying mathematical models, considering transformation matrices and DH method, for determining where the end-effector is located and how it is oriented in space, needs applying forward kinematics analysis. This process is essential for operating with industrial robotic manipulator and ensuring they can accurately perform tasks in their environment. Conversely, analysis that involves determining the necessary joint angles to achieve a specific end-effector position and orientation is inverse kinematics. It is more complex and computationally challenging than forward kinematics, especially for robots with many degrees of freedom, like the presented 6 DOF robotic manipulator. This complexity arises from the non-linear nature of the problem, potential multiple solutions, and the need to avoid points of instability.

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