APPLICATION OF MIXED-INTEGER LINEAR PROGRAMMING AND BACKWARD SCHEDULING TO REDUCE WASTE OF OPERATOR MOVEMENT

Bruna COSTA[1], Ana MOURA[2] and Liliana ÁVILA[3]

[1]*Department of Economics, Management, Industrial Engineering and Tourism, University of Aveiro, Campus Universitário de Santiago, 3810-193 Aveiro, Portugal*

[2]*Research Unit on Governance, Competitiveness and Public Policies, Department of Economics, Management, Industrial Engineering and Tourism, University of Aveiro, Campus Universitário de Santiago, 3810-193 Aveiro, Portugal, E-mail: ana.moura@ua.pt*

[3]*Research Unit on Governance, Competitiveness and Public Policies, Department of Economics, Management, Industrial Engineering and Tourism, University of Aveiro, Campus Universitário de Santiago, 3810-193 Aveiro, Portugal, E-mail: liliana.avila@ua.pt*

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ABSTRACT*: This paper presents a case study of a manufacturing company that aimed to improve the productivity of operators of the Injection Department, which is at the beginning of the production process. Efforts were directed toward optimizing the sequence of machine visits for each operator. This was achieved by applying a mixed-integer linear programming model and backward scheduling to minimize movement waste during the sequence. A preliminary analysis of the current situation was conducted to identify the tasks and movements they perform during their working hours, conduct a time and motion study, and collect data on machine autonomy. Throughout this process, various actions were implemented to address inefficiencies. These efforts led to accomplishing the target values for key performance indicators and reduced approximately 56% in the time spent moving between machines.*

KEYWORDS*: Scheduling, Backward Scheduling, Mixed-Integer Linear Programming, Case Study*

1 INTRODUCTION

In the face of increasing competition, companies must consistently aim to eliminate inefficiencies and enhance their productivity. Standardization can be a means to improve productivity by eliminating non-value-adding activities, enhancing efficiency and quality, reducing waste, and aligning processes with business needs (MÍkva et al., 2016; Mor et al, 2019).

This paper presents a case study of a manufacturing company aiming to enhance the productivity of its Injection Department operators. These operators must ensure that the machines within their sector, each with varying levels of autonomy, operate without constraints. To do that, they must visit these machines multiple times during their work shift. However, the lack of a predetermined sequence of visits often resulted in inefficiencies. Experienced operators had extensive knowledge of the machines, the molds used and their approximate autonomy. However, newer operators lacked this knowledge, making a predefined and standardized sequence highly beneficial to streamline their daily routines. Furthermore, there was no documented record of the tasks, movements, and standard times associated with the operators' activities.

The work focused on optimizing operators' visits to the machines, considering their autonomy and seeking to reduce unnecessary movements and enhance efficiency. For that, a time and motion study were conducted to identify all the tasks and movements performed by these operators during the work shift and determine standard times. The levels of autonomy of molds were also calculated and analyzed. During this preliminary work, some improvement opportunities were identified and implemented, contributing to improving some key performance indicators (KPIs). Then, a solution was developed by applying a mixed integer linear programming model and backward scheduling to determine the best sequence of visits, which minimizes the early and late visits.

Backward scheduling is a method where tasks or operations are scheduled starting from a defined end point and working backwards to determine start times (Pinedo, 2005). It focuses on optimizing the sequencing and timing of operations, and it is a versatile technique applied across various industries to optimize scheduling, reduce costs, and improve efficiency. Its applications range from project management and manufacturing (Tormos & Lova, 2003) to logistics (Li et al., 2025) and industrial scheduling (Lee & Kim, 2020), demonstrating its broad utility in complex scheduling environments. Mixed-integer linear programming is considered effective for solving various scheduling problems (Chansombat, Pongcharoen, & Hicks, 2019; Cheng & Huang, 2017; Floudas & Lin, 2005).

The reported case study can guide other organizations facing similar challenges, providing important insights into operationalizing a solution to optimize the sequence of visits to machines. It also provides an example of the application of backward scheduling to a different context, as it is often used to sequence tasks on machines.

The paper is structured as follows. First, the problem is discussed in greater detail. This is followed by a comprehensive description of the procedures implemented to develop a solution to optimize the sequence of visits to the machines by the Injection Department operators. Finally, some concluding remarks are provided.

2 PROBLEM DESCRIPTION

The company under analysis specializes in bathroom solutions, focusing on producing flush cisterns. The Injection Department operates with 76 plastic injection machines, organized into four sectors (Figure 1), and manages approximately 1,300 molds. At the start of the project, the department had four work teams, each composed of a team leader, four operators, and a technician. However, only three teams operate daily, with one assigned to each shift. This allows for a structured rotation among the four teams, ensuring balanced shifts and workdays while accommodating mandatory and additional rest periods.

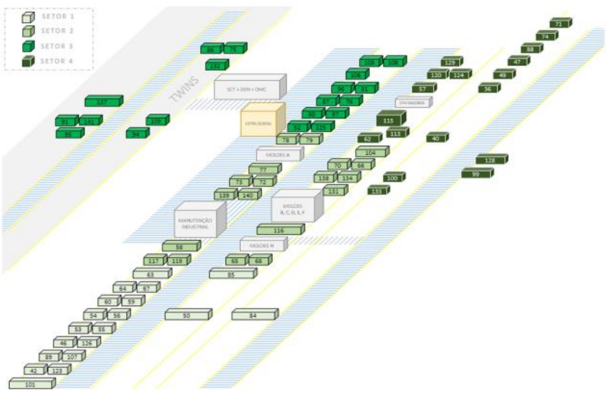

Fig. 1 Layout of the Injection Department

Operators are assigned to a single sector, each supervising a maximum of 16 operational injection machines. These machines have molds capable of producing one, two, or multiple product codes, with or without sprue—the solidified material in the plastic flow channel leading to the mold cavities—which influences the separation method. Additionally, the molds can be adapted by adding or removing inserts, modifying cavity shapes and enabling the production of distinct components.

Molds have varying cycle times, which result in different levels of autonomy. Autonomy refers to how long a mold can operate continuously without operator intervention. This means that the operator does not need to go to the machine to perform tasks, and the number of produced parts does not exceed the container's capacity. For instance, by considering the cycle time, the number of mold cavities, and the product code(s) being manufactured, it is possible to determine how many parts are produced per hour. If the container can accommodate this quantity, the mold is considered autonomous; otherwise, the operator must visit the machine before.

These workers are responsible for ensuring the efficient operation of the machines, which includes maintaining production quality, keeping the equipment clean, creating batches, and assisting with mold changes, among other tasks. However, none of these tasks followed a standardized process, and there was no recorded data on the time spent on tasks or the specific movements operators performed.

Since each mold has a different level of autonomy and mold changes frequently occur throughout the day, operators struggled to determine the optimal timing to go to each machine. Experienced operators relied on their experience to assess mold autonomy and establish a sequence for machine visits. However, newer operators often faced challenges, increasing the likelihood of arriving too early or too late at a machine. As a result, each operator visited the machines randomly in every round, choosing a path based on personal experience or intuition.

This created a need to create a solution to define the optimal sequence of machine visits for each operator to ensure greater efficiency and responsiveness.

3 DEVELOPMENT OF A SOLUTION FOR OPTIMIZING THE SEQUENCE OF VISITS TO MACHINES

To develop a solution for optimizing the sequence of visits to machines by operators in their respective sectors, an initial analysis of the current situation was conducted through daily observation of operators. The aim is to understand the tasks and movements performed during their shifts and conduct a time and motion study. Through the analysis of the data collected, several inefficiencies that hindered operator productivity were identified, leading to the proposal and implementation of improvements in task allocation. During this phase, the autonomy values of the various molds used in injection machines were also calculated.

Subsequently, a mixed-integer linear programming model combined with the backward scheduling technique was applied to define the optimal sequence of visits to the machines for injection operators during their shifts. A study was performed to establish the best rules to prioritize visits in the backward scheduling process. The mathematical model was then developed and implemented, and its effectiveness was evaluated through a pilot test.

3.1 Time and motion study and machine autonomy analysis

To address the identified challenges, a study was conducted to analyze operators' tasks, movements, and the time spent on each activity over an eight-hour work shift. Before initiating the timing process, the first step was to shadow an operator throughout their shift to observe and identify their tasks and movements. Once most tasks and movements were listed, the time study was initiated using a continuous timing method. The same tasks performed by different operators within the same sector were timed to ensure data accuracy. Additionally, the study included all sectors over eight hours (i.e., the equivalent of a full work shift), providing a comprehensive understanding of operational patterns.

As more than 200 tasks and 30 movement patterns were identified, these were categorized for a more simplified analysis. Thus, nine categories were established: IT, Recycling, Quality, Machine Autonomy, Movement/Transportation, Setup, Alarms, Cleaning/Organization, Communication, and Others. This classification allowed a clearer understanding of the time operators dedicated to each group of tasks (Figure 2). The analysis revealed that operators spent the most time on Movement/Transportation (29%). This means that, out of an eight-hour shift, approximately two and a half hours were spent on movement, representing a great opportunity for improving operator productivity.

This data collection and analysis quickly revealed two key opportunities to reassign tasks previously performed by these operators that could be performed by others. This strategic redistribution allowed for significant productivity gains, enabling operators to focus more efficiently on their core responsibilities.

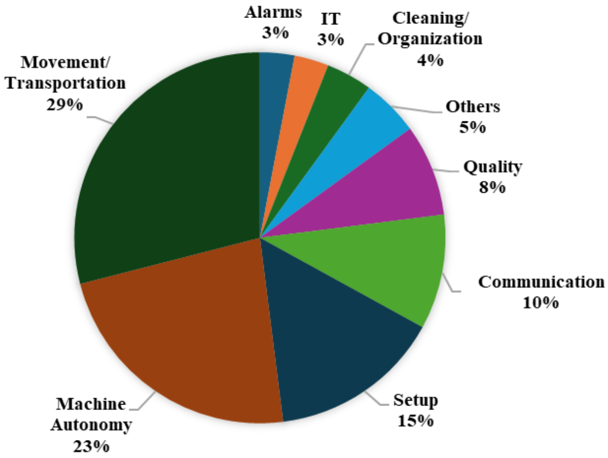

Fig. 2 Percentage of time spent daily per task group

The first opportunity focused on transporting storage cages with finished products from three injection machines to the shipping warehouse. A more detailed analysis revealed that the operator made that 15 to 16 times per shift, consuming approximately one hour of the available eight-hour work period. Recognizing this, the opportunity arose to free up this time for other activities by reassigning the task to the logistics team, which was already responsible for collecting the remaining parts produced in the Injection Department.

Additionally, the mold change process was performed by a single technician from the working team, assisted by the operator in the sector where the change occurred. Since multiple mold changes could occur within the same sector, the operator had less time to complete other tasks, and the mold change plan was not fully executed often. Only one technician per shift was responsible for various critical tasks, including mold changes, alarms, and transporting molds to the workshop, which led to recurring delays and inefficiencies.

To eliminate the time operators spent on setup tasks, which were previously identified as the third most time-consuming task group, three technicians were permanently assigned to the first shift to handle setup operations. Mold changes now take place exclusively during this shift, with technicians fully responsible for the process, eliminating the need for operator involvement. This adjustment immediately improved the achievement of target values across several KPIs, including Technical OEE (Overall Equipment Effectiveness), Overall OEE, Non-Conformities and Setup Time. Before implementation, most of these target values were not being met; however, after the change, all four indicators successfully reached their target values. Table 1 presents the values for those indicators before (January 2023) and after the implementation of this measure (February 2023).

Table 1. Evolution of KPI values

KPI	Target value 2023	Jan 2023	Feb 2023
Technical OEE	90%	88.1%	92.6%
Overall OEE	85%	74.7%	88.4%
Non-conformities	0.5%	0.5%	0.4%
Setup time	26 min	27.3 min	23.5 min

To develop a tool that optimizes the sequence of machines, an operator should visit and operate, mold autonomy was calculated. As previously mentioned, mold's autonomy can be determined by considering the cycle time, the number of mold cavities, the product code(s) being manufactured, and the capacity of the container used to store the final product. With this data collected, the autonomy of each mold was calculated for the different product codes, which were subsequently classified according to the autonomy values. Since an operator's round takes approximately one hour, all product codes with autonomy equal to or greater than 90 minutes were classified as having 'Ideal autonomy' values, and the codes with an autonomy range from 60 to 90 minutes, inclusive, were classified as having 'Reasonable autonomy'. Two more categories were defined for products requiring greater attention. Those with 'Critical autonomy', ranging from 30, inclusive, to 60 minutes and 'Highly critical autonomy', ranging from 0 to 30 minutes, indicating an urgent need for intervention.

This analysis revealed that approximately 61% of the codes fell into the Ideal and Reasonable categories. However, 39% were classified as having 'Critical autonomy' (18%) or 'Highly Critical autonomy' (21%), indicating a significant portion of products that require closer monitoring and intervention. To address this issue, a Mold Delivery Review meeting was established. Each day, based on ongoing production, three or four molds are selected and assessed according to a predefined set of criteria, ensuring a structured and systematic approach to improvement.

3.2 Optimizing the sequence of visits to machines

3.2.1 Selecting priority rules

The backward scheduling technique involves planning operations in reverse order, starting with the final operation and working backward to the first, while adhering to the established priority rules (Pinedo, 2005). For that, an initial test was conducted to determine the combination of rules to be used. This involved analyzing the machines operating on a specific date and time in sector 1. Data was collected on the machines, the items being produced, and the molds used, along with the previously determined autonomy values. Four rules – Early/Late Due Date and Shortest/Greatest Distance Between Machines – were paired, resulting in eight possible combinations.

Since two of the four analyzed rules involved the distance between machines, a distance matrix was required. A scale from one to five was established to achieve this, where one represents the shortest proximity and five indicates the greatest separation between machines.

Furthermore, it was important to account for the fact that the machines are already running when the operator begins their shift. Therefore, considering the full autonomy of each machine as the initial value would not be realistic. As such, in the first iteration, all machines were assumed to start the shift with half of their autonomy value. In subsequent iterations, the full autonomy of each machine was used.

This simulation showed that selecting Late Due Date as the primary priority rule and Greatest Distance Between Machines as the secondary priority rule yielded the best solution, with only 15 delayed tasks, 8 fewer than the combination of Early Due Date and Greatest Distance Between Machines, which had the worst solution with 23 delayed tasks. Therefore, the first combination was selected for the subsequent stages.

3.2.2 Mathematical model

After selecting the priority rules, a mixed-integer linear programming model was developed, incorporating key assumptions related to sequencing problem resolution and the application of backward scheduling.

According to this scheduling technique, each job j (j=1, 2, …, n) becomes available for processing at time zero, has a processing time p_j, and ideally should be completed by a predefined due date d_j (Low, Li & Wu, 2016). These jobs are indivisible, meaning they must be completed once started, and no additional jobs can be introduced during production. Similarly, the machine can only process one job at a time. The objective function is to minimize both the total tardiness and earliness of job completion, while striving for precise adherence to the predefined due date. However, in practice, job j may be completed either ahead of schedule or past its due date d_j, leading to an earliness or tardiness penalty, respectively (Cheng & Huang, 2017).

Building upon the mathematical formulation used to implement backward scheduling, necessary adaptations were made for the case under study. As previously mentioned, each operator is responsible for managing a set of machines from a sector, ensuring that parts are produced according to specifications, organizing and storing them properly, and maintaining a clean and structured work environment. Additionally, operators must identify scrap containers and pallets, conduct quality checks every three hours, and perform other tasks. To efficiently complete these tasks, the operator visits the machines several times. The route is determined by considering the autonomy of the molds being produced at each machine, as it directly influences the sequence the operator must follow. Based on these considerations, the symbol definitions are presented as follows:

- M: An extremely large positive integer.
- n: Number of visits.
- m: Number of machines.
- i: Machine i, where $i = 1, 2, \ldots, m$.
- j: visit j, where $j = 1, 2, \ldots, n$.
- C_j: Completion time of visit j.
- p_{ij}: Processing time of visit j on machine i.
- E_j: Early completion time of visit j.
- T_j: Delayed completion time of visit j.
- d_j: Due date of visit j (i.e., machine autonomy).

The mathematical formulation is given in (1)-(10), where the objective function (1) minimizes the total completion time of all visits, considering earliness and tardiness. Constraints (2) - (5), and (10) are general constraints, similar to those found in other single-machine sequencing problems. Constraints (2) and (3) ensure that each visit has one immediate successor and one predecessor. Constraint (4) prevents cycle formation, while constraints (5) and (10) define the variable domain. The remaining constraints are specific to backward scheduling. Constraints (6) and (7) define visit delay and early completion, and constraints (8) and (9) ensure that their values can never be negative.

$$\text{Minimize} \quad min \sum_{j=1}^{n}(E_j + T_j) \quad (1)$$

Subject to:

$$\sum_{i \in N \cup \{0\}, i \neq j} x_{ij} = 1;\ \forall j \in N \cup \{0\} \quad (2)$$

$$\sum_{j \in N \cup \{0\}, j \neq i} x_{ij} = 1;\ \forall i \in N \cup \{0\} \quad (3)$$

$$C_j \geq C_i - M + (p_j + M)x_{ij};\ \forall i \in N \cup \{0\}, \forall j \in N \quad (4)$$

$$C_j \geq 0;\ \forall i \in N, C_0 = 0 \quad (5)$$

$$T_i \geq\ C_i - d_i;\ \forall i \in N \quad (6)$$

$$E_i \geq\ d_i - C_i;\ \forall i \in N \quad (7)$$

$$T_i \geq\ 0;\ \forall i \in N \quad (8)$$

$$E_i \geq\ 0;\ \forall i \in N \quad (9)$$

$$x_{ij} \in \{0, 1\};\ \forall i \in N \cup \{0\}, \forall j \in N \cup \{0\} \quad (10)$$

The mathematical model was implemented and tested using the IBM ILOG CPLEX Optimization Studio software.

3.2.3 Pilot test

A pilot test was conducted in the real-world setting to assess the results obtained from the developed model. The sequence generated by the model was followed, with the operator being monitored while every task and movement was timed throughout the execution of the defined sequence. Table 2 presents the results, comparing two scenarios: one where the sequence obtained through the developed model was followed, and one where it was not considered. For the comparison, it was assumed that a shift consists of 480 minutes, during which the operator needed to visit 16 machines and complete 26 rounds, considering the machine with the shortest autonomy. Consequently, the time spent in movement during a shift was determined based on the duration of a single round and the total number of required rounds.

Table 1. Results of the pilot test

Measure	Using Backward Scheduling	Not using the Backward Scheduling
Duration of a single round (min)	16	36
Time spent in movement during a shift (min)	416	936
Time remaining (min)	64	-456

Under the same circumstances, it was determined that implementing backward scheduling saved 20 minutes per round. In this scenario, the operator would have 64 minutes to perform other tasks. In contrast, without the developed solution, the time available per shift would not be sufficient to complete the 26 rounds. The obtained sequence reduced the time spent moving between machines by approximately 56%. This improvement enhances working conditions, making the job more efficient and less physically demanding. It allows the operator to spend more time between rounds on other tasks, such as autonomous maintenance activities, without neglecting machine operations.

4 CONCLUDING REMARKS

This paper details the efforts to eliminate movement waste and improve the productivity of operators in the Injection Department of the manufacturing company under study. Since their work marks the start of the production process, their performance directly influences the efficiency of the assembly lines and the timely delivery of the final product to the customer.

Beyond the immediate effects of the implemented measures on KPIs and the reduction in operator movements, the enhanced productivity and greater availability of operators for critical tasks, particularly those focused on quality, are expected to bring additional benefits to the company. These include fewer stock errors, faster identification of defects or issues, and a higher quantity of produced parts. As a result, production flow will improve, especially in the assembly phase, since the parts produced in the injection department are used to supply the company's assembly lines. Ultimately, this leads to a more efficient process and a quicker response to customer demands.

To ensure that the improvements made are sustainable, it will be necessary to continue monitoring and evaluating the department’s operations and identifying new opportunities for improvement. Continuing the daily Mold Delivery Review Workshop will be important as it will enable, in the medium term, an analysis of factors such as the autonomy and separation method for a larger number of molds. This will impact the number of rounds the operator must do, the resulting sequence, and their productivity. Moreover, the proposed solution included only one sector. Extending it to other sectors will help standardize the operators' work and facilitate the integration of new employees. The developed solution has the potential to evolve into a dynamic version that updates the visit sequences in real time based on data imported from the production management system. This adaptation would enhance responsiveness and ensure greater efficiency in operational planning.

The case study presented in this paper offers valuable insights for companies facing similar challenges. It can help establish a procedure to tackle similar scenarios and identify the data needed to collect and analyze. Additionally, it provides valuable guidance on leveraging mixed integer linear programming and backward scheduling to optimize the sequence of machine visits while considering machine autonomy constraints.

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